Modeling, Simulation and Learning for Urban Crime Dynamics

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Motivation

Crime distributions exhibit **low regularity** and complex **spatio-temporal** patterns, with different crime types displaying distinct dynamics. Mathematical models provide quantitative tools for understanding these patterns and helping municipalities develop **more effective safety policies**. We focus on residential burglaries with three main objectives:

- **Modeling:** develop agent-based and continuum (PDE) descriptions.
- **Simulation:** design a robust finite element solver for realistic urban geometries.
- **Learning:** infer model parameters and latent dynamics from data.

Agent-based Model

Spatial site $s = (i, j) \in \Omega \subset \mathbb{R}^2$ with spacing h and time step Δt. Agents: **houses** (B), **burglars** (n), **delayed perception** (H), and **patrol** (m).

Dynamic attractiveness (repeat and near-repeat victimization) [4, 1, 2]:

$$B_s(t+\Delta t) = \Big[(1-\eta)B_s + \tfrac{\eta}{4}\sum_{s'\sim s} B_{s'}\Big](1-\omega\Delta t) + \theta E_s(t).$$

Crime probability under control [2]:

$$p_s^c(t) = 1 - \exp\big(-A_s(t)\, e^{-\beta m_s/h^2}\Delta t\big), \qquad A_s(t) = A_s^{\mathrm{st}} + B_s(t).$$

Criminal dynamics (biased random walk driven by A_s) [2]:

$$n_s(t+\Delta t) = A_s \sum_{s'\sim s} \frac{n_{s'}(1-p_{s'}^c)}{\sum_{s''\sim s'} A_{s'}(t)} + e^{-\beta m_s/h^2}\Gamma(1-\Sigma)\Delta t,$$

Delayed Perception of Crime [2]:

$$H_s(t+\Delta t) = \Big(1 - \tfrac{\Delta t}{\tau}\Big)H_s + \tfrac{\Delta t}{\tau}S_s, \qquad S_s = \frac{n_s p_s^c}{h^2\Delta t}.$$

Controled dynamics (biased random walk driven by H_s) [2]:

$$m_s(t+\Delta t) = H_s(t)\sum_{s'\sim s}\frac{m_{s'}(t)[1-p_{s'}^p(t)]}{\sum_{s''\sim s'} H_{s'}(t)} + m_s(t)p_s^p(t), \quad p_s(t) = 1 - e^{-\Sigma n_s(t) p_s^c(t)}.$$

Note: $\sum_s m_s = M$ is conserved.

Key parameters: η (neighborhood effects), ω (decay), θ (victimization boost), Γ (criminal generation rate), $E_s = n_s p_s^c$ (expected burglaries), τ (delay), Σ (arrest probability).

PDE Model (Continuum Limit)

Taking $h, \Delta t \to 0$ with $\frac{h^2}{\Delta t} = D$, $\theta\Delta t = \epsilon$, and non-dimensionalizing:

$$\frac{\partial A}{\partial t} = \nabla\cdot\big(\eta\nabla(A - A^{\mathrm{st}})\big) - A + \rho A e^{-\pi} + A^{\mathrm{st}}, \qquad \frac{\partial H}{\partial t} = \frac{1}{\tau}\left(\rho A e^{-\pi} - H\right),$$

$$\frac{\partial \rho}{\partial t} = \nabla\cdot\Big(\nabla\rho - \frac{2\nabla A}{A}\rho\Big) - \rho A e^{-\pi} + \frac{\Gamma\theta}{\omega^2}(1-\Sigma)e^{-\pi}, \qquad \frac{\partial \pi}{\partial t} = \nabla\cdot\left(\nabla\pi - \frac{2\nabla H}{H}\pi\right),$$

$$\nabla A\cdot\mathbf{n} = \nabla\rho\cdot\mathbf{n} = \nabla\pi\cdot\mathbf{n} = \nabla H\cdot\mathbf{n} = 0, \quad \text{on } \partial\Omega.$$

Here ρ (criminal) and π (patrol) are densities. For no patrol-control dynamics, we set

$$\pi = H = \Sigma = 0, \quad \text{for all } (t, \mathbf{x}) \in [0, T]\times\Omega,$$

leading to only two equations for A and ρ.

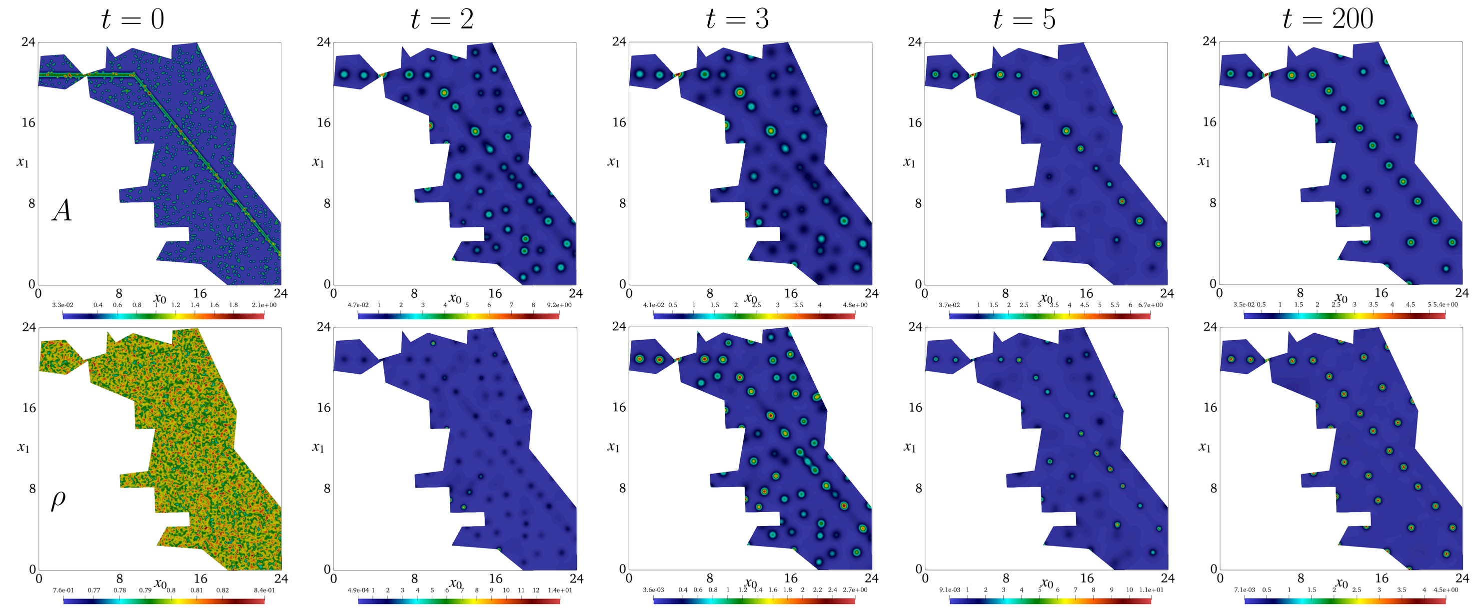
- Three free parameters: η, τ, and $\frac{\Gamma\theta}{\omega^2}(1-\Sigma)$.
- An iterative partitioned finite element scheme.
- Code: `github.com/baolihao/UrbanCrime-Sim`.
- Full algorithm, convergence and robustness study in [1, 2].

Paper [1]:

Simulation: Hotspot Dynamics

No control: Chicago with Highway I-90.

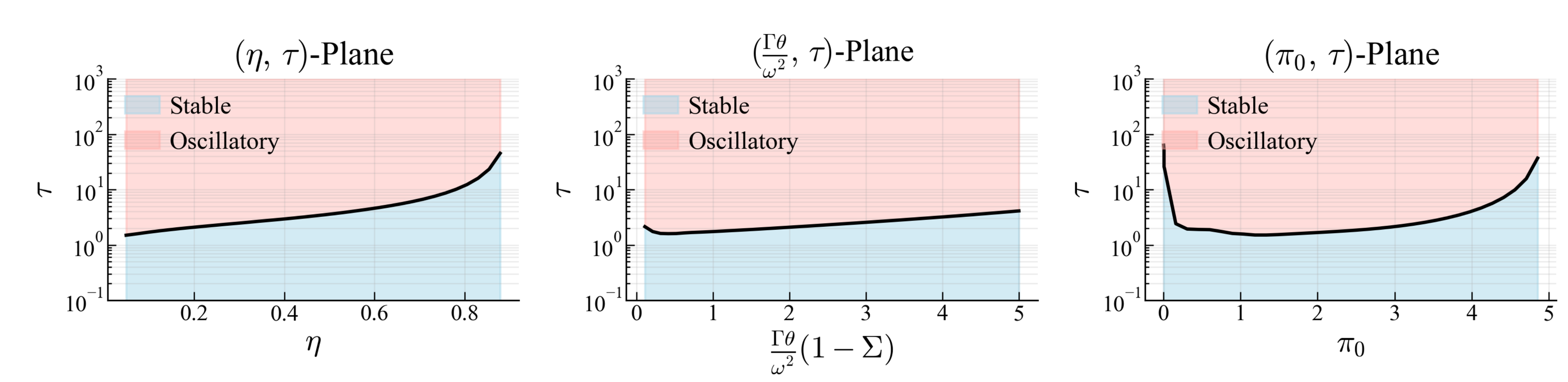
Figure 1. Evolution of attractiveness A (top) and density ρ (bottom), consistent with findings in [7].



With control.

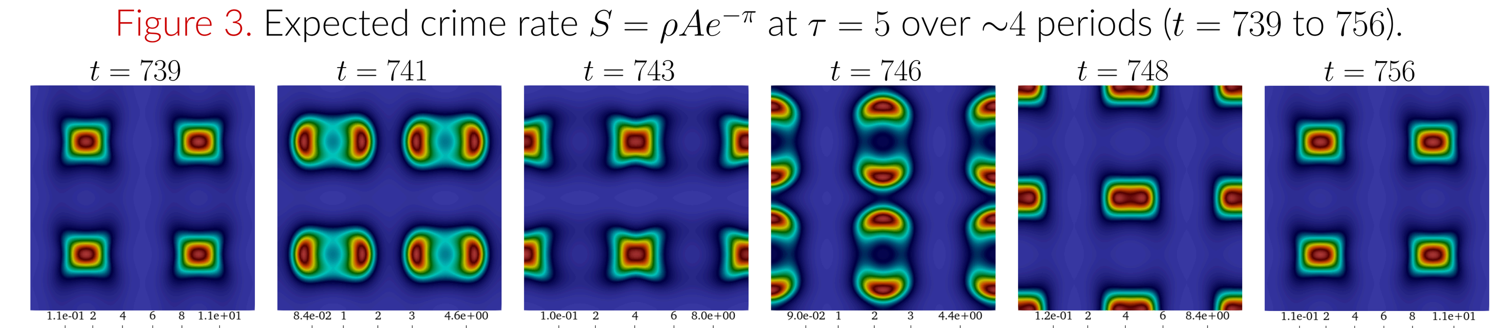
- Hopf bifurcation.
  (i). Larger η raises τ_c^*. (ii). $\tau_c^* \to \infty$ as $\pi_0 \to 0$ (no patrol, no oscillation)

Figure 2. Stability phase diagrams from the Routh-Hurwitz criterion with the black curve giving the Hopf threshold τ_c^*. **Sustained oscillations** for $\tau > \tau_c^*$, **stable** below τ_c^*.



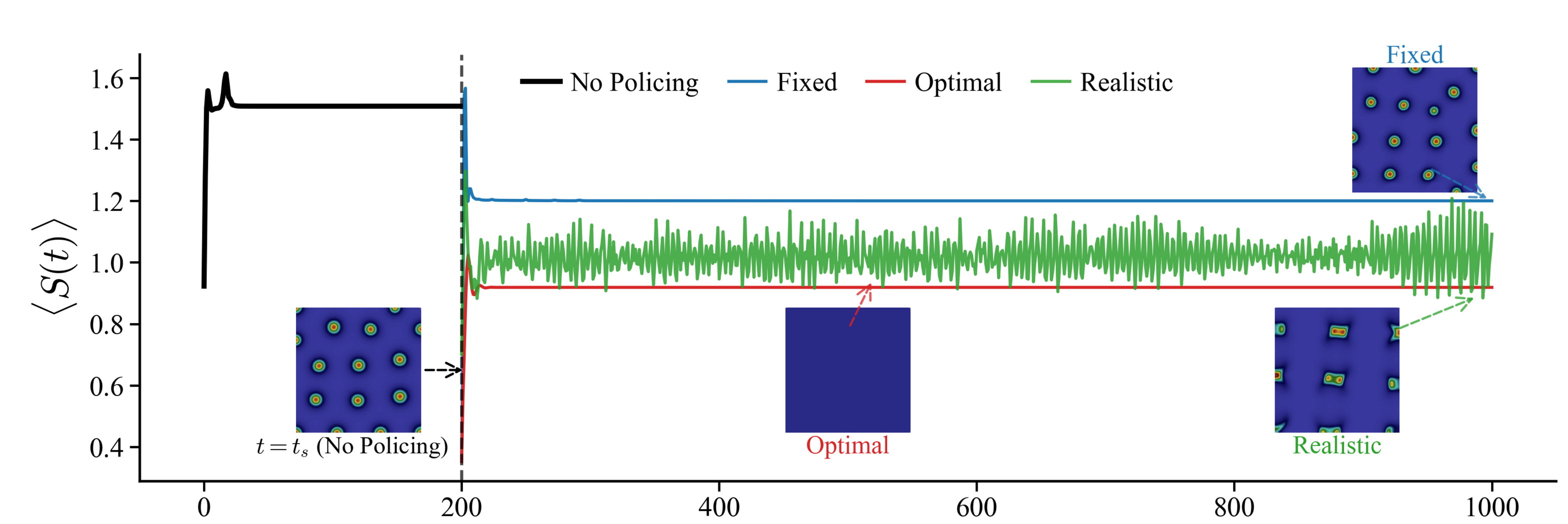
- Hotspots cyclically split, migrate, reform.
  No control → steady hotspots. With control → oscillating hotspots.

Figure 3. Expected crime rate $S = \rho A e^{-\pi}$ at $\tau = 5$ over ~4 periods ($t = 739$ to 756).



- Bridging two extremes.
  Fixed patrolling [5]: **Displaced hotspots.** Deploy once at t_s, then frozen.
  Optimal patrolling [6]: **Uniform hotspots.** Instantaneous deployment.
  Realistic patrolling ($\tau = 5$) [2]: Moving hotspots. Sustained oscillations.

Figure 4. Spatially averaged crime rate $\langle S \rangle$ for three policing strategies: Deploy at t_s, $M = 50$, $\eta = 0.15$.



Recover Hidden Dynamics: Self-Test [3]

Observed data. From noisy observations $A(t, \mathbf{x})$, $\rho(t, \mathbf{x})$, $\pi(t, \mathbf{x})$, $H(t, \mathbf{x})$, we recover the spatially varying fields

$$\eta(\mathbf{x}), \qquad q(\mathbf{x}) := \frac{\Gamma\theta}{\omega^2}(1-\Sigma), \qquad \tau(\mathbf{x}), \qquad A^{\mathrm{st}}(\mathbf{x}).$$

Define $C := \rho A e^{-\pi}$, $\mathbf{J}_\rho := \nabla\rho - 2\rho\nabla\log A$, and $\mathcal{G} := A^{\mathrm{st}} - \nabla\cdot(\eta\nabla A^{\mathrm{st}})$. Then the controlled equations used for learning become

$$A_t = \nabla\cdot(\eta\nabla A) - A + C + \mathcal{G}, \qquad \rho_t = \nabla\cdot\mathbf{J}_\rho - C + q e^{-\pi}, \qquad \tau H_t = C - H.$$

Weak-form learning : Express

$$\eta = \sum_j \alpha_j\psi_j, \quad \mathcal{G} = \sum_j \gamma_j\psi_j, \quad q = \sum_j \beta_j\psi_j, \quad \tau = \sum_j \delta_j\psi_j.$$

Test functions $\varphi_i(t, \mathbf{x})$ vanishing at $t = 0, T$, then integration by parts gives

$$\sum_j \alpha_j \iint \psi_j\nabla A\cdot\nabla\varphi_i - \sum_j \gamma_j \iint \psi_j\varphi_i = \iint A\,\partial_t\varphi_i - \iint (A - C)\varphi_i,$$

$$\sum_j \beta_j \iint \psi_j e^{-\pi}\varphi_i = -\iint \rho\,\partial_t\varphi_i + \iint \mathbf{J}_\rho\cdot\nabla\varphi_i + \iint C\varphi_i,$$

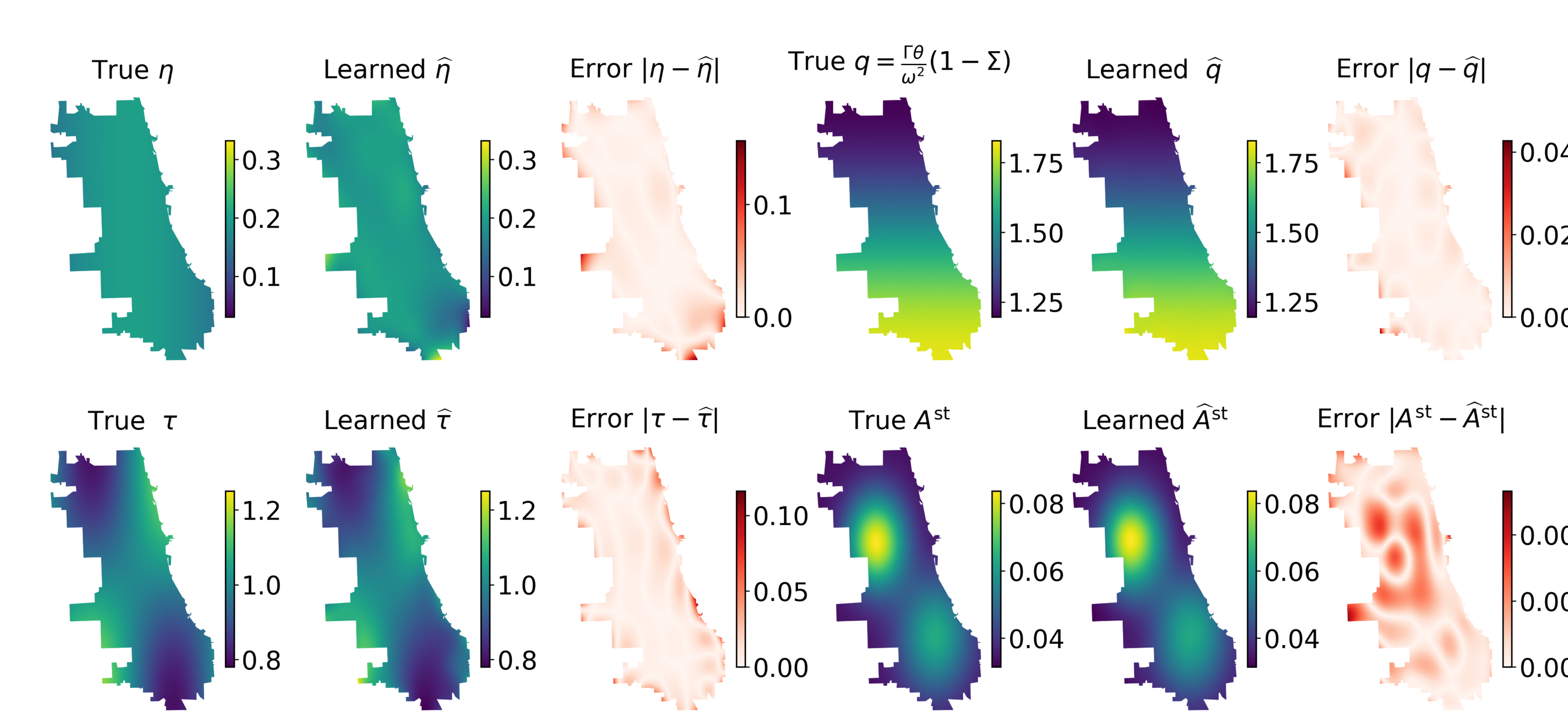
$$\sum_j \delta_j \iint \psi_j H\,\partial_t\varphi_i = \iint (H - C)\varphi_i.$$

Solve for the coefficients, then recover A^{st} from

$$A^{\mathrm{st}} - \nabla\cdot(\eta\nabla A^{\mathrm{st}}) = \mathcal{G}.$$

Parameter Recovery: Results

Figure 5. Spatial parameter recovery on the Chicago geometry. Full-domain relative L^2 errors of 7.84% for η, 0.17% for q, 1.10% for τ, 1.19% for A^{st}.



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